Co-funded by the European Union

STEAM IN TIMES

Manipulating math through history

GUIDE

Училище „Любен Каравелов" Видин

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STEAM IN TIMES

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Chapter 0 : Introduction

Welcome to the STEAM in Times project!

We believe that the materials developed and tested in this project can help younger students acquire competencies in STEM subjects as well as mathematics and critical thinking skills, which are necessary for solving problems and making decisions. With the addition of the artistic component of STEAM education, students think more creatively, which is the right way to generate new ideas and innovations. STEAM education generally provides a strong foundation for future learning in these fields and can help students develop a lifelong interest in them. STEAM education is especially important in today's fast-paced, technology-driven world. Early exposure of students to these subjects will result in better preparedness for the demands of the future.

Through this project we provide a series of tools for teachers to offer a cross-curricular experience to students that will help understand the evolution of mathematics and its importance, but also to get acquainted with mathematical concepts.

The target group is mainly primary education students, teachers, educators, and education professionals. It is especially important to reach students in early age on in order to avoid math anxiety specially for students with specific learning disorders (SLD) while using historical situations to contextualize mathematics. This project also targets third parties such as parents for example because the activities can be reproduced at home between parents and children in an enjoyable learning experience.

This guide is pedagogical tool. It is meant to be practical and user friendly, with clear structure and concrete in-situation explanations to facilitate daily use by the target groups.

Special attention will be given to make this guide and all the other materials provided inclusive and user-friendly for students with Specific Learning

Disorders (SLD) but also for any student that is part of the groups most likely to fall behind in STEAM subjects, for example students with fewer opportunities.

In developing materials in this project, three partners are involved. Two schools: Elementary school Lovre pl. Matacica in Zagreb, Croatia and Osnovno uchilishte "Lyuben Karavelov" in Vidin, Bulgaria. Both schools have good foundations in the form of educated teachers and equipment for teaching subjects within the STEAM area. The third partner Logopsycom, Mons, Belgium had several projects in the past regarding STEAM education, with a particular focus on the needs of students with SLDs.

This guide consists of the following chapters:

- Results of mathematics education in Bulgaria, Belgium and Croatia where the achievements of students on international tests are described, as well as the context of teaching mathematics in partner countries.
- STEAM challenge in which the difference between STEM and STEAM is described, what are the advantages of teaching STEAM, and the importance of applying the skills and attitudes that are acquired by teaching the arts and humanities.
- Discovering European cultural heritage through STEAM in mathematics classes is a chapter with description of which mathematical concepts can be represented by facts from European Historical Heritage, how to approach and why is it important to do it in primary schools as it is target audience.
- Implementation of STEAM learning through pedagogical manipulations is a chapter with an explanation of the benefits of using manipulations and what is considered a manipulation.
- Project activities is a chapter which brings in focus preliminary preparation, description and structure of drawings, materials, exercises and 3D projects included in STEAM lessons. A few examples and

illustration of how to design the manipulation and apply it in mathematics lessons can be found here.
- Conclusion offers some results achieved with students, how did the work with manipulations impact students learning, interest and motivation and also about collaborative work with parents.

Our job as teachers is to provide all students a strong foundation in STEAM area. As the world is rapidly changing, and many of the jobs of the future require skills in science, technology, engineering, arts, and mathematics, broad STEAM education can help to prepare them for the demands of the future. We hope that materials developed through this project, as well as this Guide will help teachers to integrate manipulations in their lessons and thus enable all students to understand and adopt mathematical concepts more easily. It is also important for students to understand that even in the past, even without today's technology, people managed to make many valuable things that today belong to the historical heritage. Today's achievements in technology, science, construction, medicine, communication and other fields are based on STEAM knowledge from the people that lived centuries ago.

So let's get started, have fun and learn!

Chapter 1 : Mathematics learning outcomes in Bulgaria, Belgium, and Croatia.

1. Mathematics learning in Bulgaria

PISA examines functional literacy levels in schools in 79 countries. 600 thousand students participated in the study, 199 schools and 5294 students from Bulgaria took part.

The latest survey in 2018 focused on reading comprehension, with math and science having a lower weight in the measurements. Bulgaria recorded the second largest decrease of all 79 countries covered by PISA. According to the data, 31.9% of Bulgarian students did not reach the minimum second level in any of the three indicators, making them functionally illiterate.

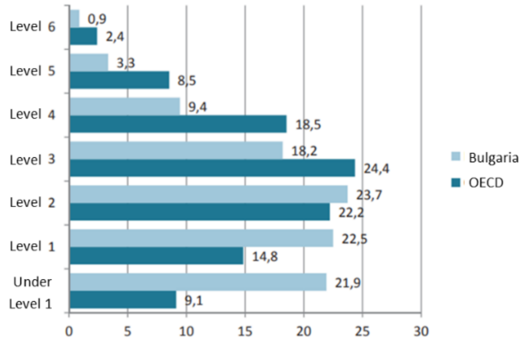


Figure 1 Source: 28.04.2023 - text_Pisa_2019.indd (government.bg) The original graphic is in Bulgarian. The titles here are translated into English by the team.

In mathematics the number of Bulgarian students who do not reach level 2 (44.4%) is almost twice the OECD average (23.9%). 4.2% of Bulgarian 15-year-olds reached the highest levels of the mathematics scale in PISA 2018.

According to the latest edition of the Programme for International Student Assessment (PISA) in 2018, conducted by the Organisation for Economic Co-operation and Development (OECD), Bulgaria showed lower results in the field of mathematics compared to the average scores of other participating countries.

Here are some key observations from Bulgaria's results in PISA 2018 in the field of mathematics:

Average score: Bulgaria achieved an average score of 437 points in mathematics, which is lower than the OECD average score (489 points).

Ranking: Bulgaria ranked lower compared to the average value of other countries, occupying the 47th position out of 79 participating countries.

Differentiation: There are significant differences in achievement among students in Bulgaria. Approximately 11% of students demonstrated high performance, while around 28% of students showed low performance.

Gender gap: The results show a slight gender gap, with boys performing slightly better than girls in mathematics, but the difference is not significant.

These results point to the challenges facing the educational system in Bulgaria in mathematics education. It is important to focus on improving the quality of mathematics education and promoting higher achievement among students. There is a need to support the development of more active and practical approaches to mathematics, which motivate students to apply mathematical concepts in real-life situations and develop analytical thinking and problem-solving skills.

The performance of Bulgarian students in mathematics has improved slightly over the years, mainly between 2006 and 2012. In real terms, achievement is at a stable but low level compared to the OECD average of 500 points.

More and more schools around the world are looking at ways to integrate STEAM into the educational environment. In Bulgaria, STEAM education has not yet been widely implemented and is mainly associated with the introduction of STEAM programmes in individual educational institutions.

At the primary level of education in Bulgarian schools, there are trends to break the traditional model of teaching. The main factors for change are the development of the network of innovative schools and the introduction of the STEAM approach in education. The focus is changing towards integrated cross-curricular interaction with a practical, results-oriented focus.

2. Mathematics learning in Belgium

According to Belgium's last PISA results, Mathematics scores in the Wallonia-Brussels Federation (495) are up slightly and now above the OECD average (489). This score is lower than the score of Flanders. Science results (483) are stable compared to previous cycles and not significantly different from the OECD average (485) (Université de Liège, 2019). As for reading, the Wallonia-Brussels Federation remains below the OECD average of 487 with a score of 481, left behind here, too, by Flanders (502) (The Brussels Time, 2019).

Education in the Wallonia-Brussels Federation (the French-speaking part of Belgium) has recently undergone many changes. Amongst these, we can note the arrival of the common core, which extends from the 1st year of primary school to the 3rd year of secondary school. This has naturally implied the creation of new reference frameworks. The STEAM approach is not yet fully integrated in Belgium since these subjects are separated in the new reference materials of the "Pact for Excellence in Education". Therefore, there is a mathematics curriculum, a technical, technological and digital manual training curriculum a science curriculum and another curriculum including history and geography. However, at the end of each reference framework, a large chapter is devoted to the possible crossovers between the different disciplines, which is more akin to the STEAM method.

The mathematics reference framework addresses 4 aims: solids and figures, quantities, numbers and data processing. The aims are then detailed in specific competencies. The framework recommends approaching concepts through concrete manipulations for each area to engage and motivate students.

Although the STEAM approach has not yet been integrated into curricula, since 2007 the Directorate General for Compulsory Education has been coordinating a movement to promote STEAM in education. The result is the "Sciences & Education guide for pre-school, primary and secondary schools", a document that brings together all the existing initiatives for working with STEAM. It is aimed at teachers at all levels who are looking for resources, training and ideas for STEAM teaching (Fédération Wallonie-Bruxelles, 2023).

3. Mathematics learning in Croatia

At last PISA exam (2018), Croatian students achieved a below-average score of 464 points and is in 40th place in the overall ranking of 78 countries. In a period of twelve years (PISA 2006-PISA 2018) in Croatia, no significant positive or negative trend was observed in the achievements of Croatian students in mathematical literacy.

From 2019, mathematics is taught in Croatian schools according to the new curriculum. Mathematics concepts are grouped in the domains Numerals, Algebra and Functions, Form and Space, Measurement, and Data, Statistics and Probability, which stem from the domains of the mathematical field of the curriculum.

The domains are gradually developed and built upon throughout the entire vertical structure of learning and teaching mathematics, and the share of a particular domain in a cycle is tailored according to the developmental capabilities of students and the need for systematic building of the entire mathematical education. Domains involving terms like number and form are more prominent in the lower cycles, while in the higher educational cycles the domains of more complex mathematical concepts, such as functions or probabilities, are covered in more detail. At the level of each individual year of learning and teaching, for each domain the learning outcomes are presented, which are clear and unambiguous statements regarding the expectations from students.

Chapter 2 : The STEAM challenge

1. STEM and STEAM

The main difference between STEM and STEAM goes beyond the letter A. The distinction lies in the fact that STEM solely focuses on science and places emphasis on scientific methods, concepts, and ways of proving specific claims. While STEM encompasses the fields of science, technology, engineering, and mathematics, STEAM explores the same concepts but in a completely different manner. STEAM recognizes the significance of arts and humanities as an integral part of education. By including arts alongside the traditional STEM subjects, STEAM emphasizes the importance of fostering creativity and innovative thinking within scientific endeavours. This interdisciplinary approach not only sparks curiosity but also makes STEM subjects more captivating and relatable to students. It encourages exploration and problem-solving through creative endeavours.

For instance, students working together as a team can apply their understanding of STEM concepts to design visually appealing products or objects, such as constructing bridges using architectural principles or developing a unique adhesive mixture for bonding construction materials. By integrating arts and humanities, STEAM broadens the horizons of learning and cultivates well-rounded individuals capable of embracing diverse perspectives and applying critical and imaginative thinking to real-world challenges. The focus is on teamwork, which develops and promotes collaboration, respect for others' ideas, and consideration of different team members' perspectives.

Teamwork is also essential for cultivating patience, perseverance, and mutual cooperation instead of competition. In addition to teamwork, students also develop other competencies, such as learning skills, technology usage, problem-solving, innovation, critical thinking, collaboration, adaptability, and social and cultural awareness. Such an approach stimulates research, analysis, logical thinking, reasoned debates, learning through trial and error, hands-on learning, and the integration of all acquired knowledge.

It is crucial to demonstrate and explain to young individuals why the knowledge and skills they acquire are valuable. The employees of the future must possess the ability to creatively solve problems, collaborate in finding solutions, and discover new methods.

Today, there is an increasing emphasis on the need to popularize mathematics and science in general, as we enter a new era of digitization, computerization, and robotics that requires many new solutions. However, there is significant resistance among young people in schools towards STEM subjects. One of the main reasons is that students fail to see the purpose of learning these subjects since they are mostly taught in a theoretical manner with few practical applications. Additionally, the mistaken belief that boys perform better in this field predominates. Fortunately, numerous initiatives nowadays advocate for the equal participation of both girls and boys. A different, more practical approach to STEM through STEAM, starting from concrete real-life situations that need to be solved and discovering the theory through them, can motivate and interest young people in pursuing STEAM careers in the future.

STEAM is not a new concept. It encapsulates the timeless relationship between science, technology, engineering, arts, and mathematics, recognizing the profound significance of arts and humanities in our understanding of the world. Throughout history, science and art have coexisted and enriched each other, revealing their inseparable connection. Art serves as a medium through which we can grasp complex scientific concepts, and science, in turn, becomes more accessible and captivating when approached through an artistic lens. This integration allows us to gain a deeper comprehension of our surroundings and navigate it with heightened efficacy.

The fusion of science and art has been exemplified by brilliant minds throughout the ages. Visionaries such as Nikola Tesla and Leonardo da Vinci exemplify the importance of merging science and art. Tesla's scientific breakthroughs were fuelled by his imaginative and creative thinking, while da Vinci's artistic genius informed his scientific inquiries. Leonardo da Vinci's diverse talents as a painter, scientist, and inventor showcase the transformative potential of embracing both art and science. His iconic artworks are accompanied by a vast array of inventions, ranging from architectural designs to anatomical studies, demonstrating his holistic approach to knowledge.

Art has been an integral part of the human experience since ancient times, resonating with our innate need for creative expression. Even millennia before the Common Era, early humans adorned the walls of their dwellings with cave paintings that vividly depicted their daily lives. These ancient artworks portrayed a plethora of subjects, including plants, animals, tools, and weapons, offering us a glimpse into the rich tapestry of our ancestors' existence.

Beyond its historical significance, the inclusion of arts and humanities within the STEAM framework holds immense value for contemporary education. Integrating artistic elements into STEM subjects fosters a holistic approach to learning, enabling students to develop essential skills such as critical thinking, problem-solving, and innovation. Artistic endeavors encourage exploration, imagination, and the cultivation of a unique perspective that can transcend disciplinary boundaries.

Moreover, arts and humanities nurture empathy and cultural understanding, promoting an inclusive and well-rounded education. They allow individuals to explore diverse perspectives, challenge existing norms, and engage with complex societal issues. By incorporating the humanities into the STEAM curriculum, we equip students with the tools to analyze, critique, and contextualize scientific advancements within broader ethical, social, and cultural frameworks.

Europe has a long tradition of scientific knowledge being transmitted orally from one generation to another. STEAM education in schools provides students with an opportunity for creative learning, utilizing 21st-century skills such as problem-solving.

These general skills are key to developing a future-ready workforce that understands the potential of "what if" when solving real-life problems. They also point us towards developing 21st-century skills such as civic and cultural engagement, economic productivity, lifelong learning, managerial competencies, and communication.

Teaching STEM subjects is crucial because science is all around us. In our daily lives, technology is constantly advancing, and its development is accelerating every day. That's why it is necessary to create a generation that not only skillfully uses technology but also continuously develops new solutions to help humanity. In all of this, mathematics holds invaluable value and significance as it aids in analytical thinking and solving complex problems. On the other hand, the development and role of other qualities such as creativity and critical thinking should never be neglected.

Many opportunities for hands-on learning are emerging in schools and institutes across the country. This is generally referred to as "maker spaces." These are places, whether within schools or cultural and city institutions, where young people are presented with a specific problem or theme. This type of learning encourages collaborative learning and discovering solutions, utilizing scientific and technological resources such as soft circuits, embedded video, game creation, data art, and more.

It is evident how this type of learning is entering the everyday curriculum of various educational institutions, where the context of art is used to demonstrate STEM concepts and vice versa.

Therefore, the "what if?" idea does not depend on the purchase of specific STEAM technologies or even the design of a classroom or maker space. It largely depends on the imagination and curiosity of teachers collaborating with their students.

2. Why did we choose STEAM?

We choose STEAM instead of STEM because STEAM program that incorporates arts into the curriculum has been proven to increase creativity, improve academic performance, increase motor skills, enhance visual learning, and strengthen decision-making skills. According to this we think that STEAM will be more suitable for pupils in our primary school. Art is not just work in the studio. Arts is a term that represents language arts, social studies, physical arts, fine arts, and music. Many people feel that adding an A is unnecessary and that the application of creativity and art is a natural part of STEM, but we would like to emphasize this. Art is discovering and creating ingenious ways of solving real problems, integrating principles or presenting information. Think of an engineer, they use engineering, math, technology, science and art to create stunning buildings and structures. We also think that the STEAM approach to education is essential for the development of individual skills of pupils in our primary school. Through quality education, this allows pupils to be creative and innovative from an early age (grades 1-4). STEAM education encourages pupils' imagination, inspires and motivates pupils to study and improve in areas essential for the development of society and social competitiveness, which can ultimately lead them to choose highly paid and competitive professions in continuing education after primary school. In addition, the acquired knowledge and skills can be useful to students in solving problems and challenges in everyday life. For this reason, we will often use art in our work, and in the abbreviation we will add the letter A (for the English word art).

In continuation, attention will be given to the connections between the five STEAM disciplines. This is flagged as an area requiring attention, particularly in current integrated STEAM curriculum where connections between the disciplines are largely implicit. To increase clarity in the definition of integrated STEAM and the aspects of integrated STEAM lessons that should be evaluated, we propose an instructional framework that puts accent on the depth and breadth of STEAM learning that emphasizes both the learning within individual disciplines as well the connections between the disciplines.

Learning in many disciplines has traditionally been defined by unique knowledge and practices within clearly marked parameters. As such, knowledge across different disciplines, for example the sciences and social sciences, are often thought to be different from each other . The sciences have been viewed as a field of study that is more systematic and predictable while the social sciences are perceived to be more varied and less predictable. The disciplines of science, technology, engineering, mathematics and art are traditional stand-alone disciplines with their own unique conceptual, epistemic and social constructs, at least in academic settings. In the attempt to design integrated STEAM curriculum, it is fundamental to question and examine how the traditional boundaries of each domain can be changed and if integrated STEAM can be considered an independent discipline that possesses its own unique practices and constructs.

Science as a discipline focusses on the study of the natural world that includes the laws of nature (National Research Council (NRC) (2009)). The body of knowledge in science is generated through the process of scientific inquiry and is accumulated over time. Scientific knowledge can be used to inform engineering design processes. Technology as a field is probably one that is most diffused in terms of the problems or issues unique to the discipline. Historically, technology is a system which creates artefacts that can be applied to solving problems and making life simpler. Engineering as a discipline consists of both the knowledge about design and creation of human products and developing processes of problem solving. In designing products and devising solutions to problems, engineering applies scientific and mathematical concepts together with technological tools. Art is the expression or application of human creative skill and imagination, typically in a visual form such as painting or sculpture, producing works to be appreciated primarily for their beauty or emotional power. As we can notice, art is represented in all 4 STEM disciplines. Finally, mathematics is a discipline that delves into the study of patterns and relationships among quantities, numbers, and space. The unique feature of mathematics is that claims are warranted through logical arguments based on foundational assumptions instead of empirical evidence. Hence, knowledge in mathematics is not overturned unless the assumptions on which it is built on are changed. Disciplines in STEAM are mostly strongly classified or compact discipline with unique practices, conceptual constructs and ways of thinking. In integrated STEAM, what researchers are trying to do is to blur the boundaries defining the five disciplines and apply them in such a manner that will mutually enhance the practices of individual disciplines.

In the recent two decades, learning about STEM and STEAM education has gained traction worldwide as cross-disciplinary (compared with mono-disciplinary) knowledge and skills are valued in modern times to meet the demands of the fourth industrial revolution and a world that is increasingly characterized by the blurring of disciplinary divides and technological infusion. The advent of the fourth industrial revolution places importance on digitization and technology in human life and communities. Their impact on and transformation of the lives of ordinary people have never been more significant. Yet, despite the overwhelming outputs of STEAM knowledge and artefacts, the abilities of our educators and young people to take advantage of new opportunities that the world has to offer remained diffused. The complex problems, such as climate change and cyber threats, faced by the world today require knowledge and skills from different disciplines to appreciate and understand the issues in order to live and participate in the world in a meaningful manner. Beyond preparing for the future work of our pupils in school, the need for every citizen to understand and play their part to combat the complex problems of the world provide the compelling (while pragmatic) reasons for embracing integrated STEAM education in schools.

S-T-E-M Quartet framework begins with a single lead discipline as the focus and subsequently examines how knowledge and skills of the lead discipline are connected and related to the other four disciplines.

3. Advantages

STEAM fields have played a significant role in history, and many of the great achievements of the past were made possible by the application of STEAM knowledge. Many of the most impressive structures throughout history, from the pyramids of Egypt to the Colosseum in Rome, were made possible by engineering principles and mathematics. Throughout history, art has been heavily influenced by mathematics and geometry, as well as by the technology available at the time, such as the development of perspective during the Renaissance.

Each day, engineers across the world take the principles of mathematics and science and apply them to solve real-world problems, designing and constructing everything from cars and bridges to new types of chemicals and computer software. In a professional setting, engineering is separated into six disciplines: chemical engineering applies chemistry, mathematics, biology, and physics to manufacture fuel, medicines, materials and even food products. In recent years, developments in this area have been instrumental in advancing the efficiency of renewable energy sources and increased battery capacity for electric cars.

Electrical engineering is one of the newer engineering disciplines and focuses on electronics and electrical equipment. Those working in this specialty create a wide range of products, from small objects like microchips and computer hardware to gigantic power generators and satellites that orbit the Earth.

Mechanical engineering deals with the creation and development of mechanical systems that involve any type of motion. Using the principles of math and physics, mechanical engineers design and produce many different machines including wind turbines, airplanes, cars, prosthetics and machining equipment.

Industrial engineering is used in a wide range of industries to increase quality and efficiency. Examples include developing more effective and accurate supply chains, and designing practices and strategies that enable safer work environments.

Transportation engineering is the planning, construction, and operation of systems for moving goods and people by highway, rail, air, water, and pipelines, as well as urban and intermodal transportation. The environment is a major consideration in transportation engineering.

Civil engineering focuses on building structures that are used by the public, including roads, airports and sewer systems. Some of the most famous of these structures include the Golden Gate Bridge, the English Channel Tunnel, the Great Pyramid of Giza and the International Space Station. Along with engaging in the other components of STEAM, practicing engineering allows children to tap into their natural curiosity and develop creative ways to overcome challenges they have experienced in their own lives.

We are also witnessing the exceptional development of medical technology for diagnosis and early detection of diseases. Biomedical engineers create better prostheses, implants, and robots that are very precise in demanding operations. The quality of treatment of diseases is constantly increasing. STEAM fields play a major role in medicine and its development. STEAM education fosters interdisciplinary thinking, enabling students to work collaboratively across different disciplines. This collaboration leads to new ideas, innovation, and more effective healthcare solutions in the future. Global health challenges, such as pandemics, require a multidimensional approach. STEAM education equips students with the necessary skills to understand and address these challenges. Data collection and analysis are essential for understanding diseases, identifying patterns, and developing effective treatments. The use of mathematics, statistics, and technology allows healthcare professionals and researchers to analyse large datasets, identify trends, and make data-driven decisions.

Communication has always been an important aspect of human society. In the beginning, the main form of communication was verbal communication. It was the primary way people communicated for the majority of human history. The development of writing represented a significant advancement in communication. Information, announcements, and other types of messages were sent through letters. Ancient Egyptians, Mesopotamians, and many other cultures used early forms of writing such as hieroglyphs and cuneiform. This type of communication required time for writing, delivery, and reading by the recipient.

With the development of organized postal systems, communication through letters became faster and more reliable.

The invention of the printing press and the development of printed word greatly improved communication and had an impact on many social changes. Printed materials such as newspapers, books, magazines, and flyers enabled the mass dissemination of information and ideas. People could access the same information and discuss it.

In the 19th century, the telegraph was invented, enabling fast long-distance communication through electrical signals. Over time, telegraph technology advanced, and automatic telegraphs were introduced, allowing for faster and more reliable message transmission.

This was followed by the development of telephone communication, which enabled real-time voice communication over long distances. The development of the telephone had its roots in electromagnetism and telegraphy research. Throughout the 20th century, telephone technology continuously evolv1.ed. Enhanced telephones with improved sound, smaller dimensions, and enhanced functionalities became available. Other features such as dialling numbers and telephone exchanges for call management were introduced.

The invention of radio and television significantly changed how people receive information. Mass broadcasting of various content became possible. People could listen to radio broadcasts and watch television programs that aired news, shows, movies, and other content. Over time, television experienced many technological advancements. Colour was introduced into television broadcasts, and later digital television systems were developed, offering improved picture and sound quality.

STEM (Science, Technology, Engineering, and Mathematics) and engineering disciplines have been crucial in the accelerated progress of technology and computer science. This enabled the development of digital communication tools that have become an integral part of our daily lives.

Fast broadband internet, optical cables, wireless networks, and telecommunication infrastructures have allowed us to communicate faster and more efficiently.

Email, chat, forums, social networks, and various messaging applications enable fast and global real-time communication.

Social networks enable us to connect with other people, share content, participate in discussions, and follow the latest news.

The development of STEAM (Science, Technology, Engineering, Arts, and Mathematics) has brought about remarkable advancements in communication, thanks to the integration of arts and humanities. By recognizing the essential role of arts and humanities within STEM fields, communication has become more than just a technical process. The inclusion of arts fosters creativity, enabling the exploration of new expressive modes, while the humanities provide a critical lens through which we analyze the societal and ethical implications of communication technologies. This multidisciplinary approach, emphasizing creativity, critical thinking, and a broader understanding of human experiences, has propelled the rapid development of innovative and highly effective communication methods. STEAM experts are involved in developing recommendation algorithms, analytics, and security measures on social networks.

Multimedia content such as images, videos, and animations has become an integral part of digital communication. They can quickly convey information, enhance understanding, and make communication more engaging and interactive.

Video calls and conferences via the internet have also been developed. Video calls allow people to communicate without the need for physical travel from one place to another. This is particularly useful in the business world as it enables faster decision-making, remote collaboration, and reduces travel costs.

With the advancement of mobile technology, communication through mobile phones has become accessible to almost everyone. People can send messages, make calls, browse the internet, and use applications on mobile devices, enabling constant connectivity and communication from anywhere.

There have also been advancements in language technology, such as machine translation and speech recognition. This has facilitated international communication and removed language barriers. Speech recognition technology also provides greater accessibility for individuals with physical or motor disabilities. People who have difficulties with typing or writing can use speech recognition to communicate, access information, and use technology. This can improve their ability to participate in society, education, and professional work.

STEM, as well as art and humanities within STEAM, are crucial for the development of technologies and tools that have improved and accelerated communication, enhancing our lives and enabling us to be more connected than ever before.

4. The integration of the arts and humanities in STEAM

If we look at a human being as a complex system of functions, thoughts and skills that are growing in everlasting process of learning throughout life span, we need to include self-experience in the whole equation. That is where different fields of Art (visual, music, movement, applied art and design) walk into the picture helping a person, especially young one that has noticeably short concentration, to understand and discover interest in often difficult areas of STEM: Science, Technology, Engineering and Mathematics. It is no wonder that there is a saying how "picture says more than 1000 words".

Perfect example of that type of learning was time of Rennaissance and Humanism during the 15th and 16th century with many extraordinary personalities that shine brightly even nowadays of whom Leonardo da Vinci and Michelangelo Buonarotti were true masterminds. Not just artists (painters and sculptors), they were ingenious inventors, experimenting in fields such as Architecture, Construction Industry, Anatomy, Design of different instruments for Military and Science as well as Teaching their apprentices though the workshops they led. Could that be why they developed and accomplished so much and are well remembered in the course of History? That is how introducing Art in STEM could be the answer to developing every person fulfilling their true potential. It helps students in motivation, explanation, understanding, seeing practical purpose of what is taught and build their self-confidence.

In every nation we can find inspiration with those perfect examples of ingenious individuals that can lead us through this guidebook. It will be interesting to learn from one another and implant these ideas into our future work as teachers.

The process of design thinking and its active methodology is proving to be useful for developing skills needed in the 21st century as key methods to master the challenges of an uncertain future. Methods of teaching and a variety of techniques change according to different times' needs. Nowadays we use different apps, flash cards, quizzes, visual thinking, project learning, blueprints from the past, playing motivating games, making experiments of "try and error" where we allow students to think, develop their own ideas and learn though their mistakes taking it as a legitimate process of education.

In conclusion the circle of good education consists of these few steps: empathizing with a situation, defining a problem, conceiving the answer to solve the problem, making a prototype, and testing it while assessing the solution. We can learn a lot of those steps from history while finding the solutions of today making a path for humanity into a better future.

Chapter 3 : European cultural heritage through STEAM

According to the Bulgarian poet, writer, encyclopaedist and journalist Lyuben Karavelov "The basis of all knowledge is mathematics". We are also confident that the STEAM approach will engage and intrigue our students in math classes. The connection between mathematics and history has been present in every aspect of life since the dawn of civilization.

STEAM-based learning is essential for today's professions as well as the professions of the future, as real-world jobs require interdisciplinary learning in which different science disciplines integrate and work together.

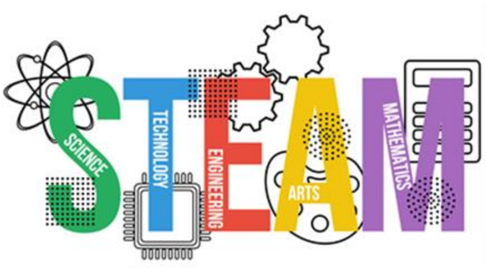


Figure 2 Source: 16.05.2023, From STEM to STEAM – DDC Engineering Solutions

STEAM sciences and their in-depth study in school play a leading role for future achievers and their educational and professional development and indirectly for all areas of society. Students can have a higher interest and learn more effectively when they learn from each other and their peers and when students themselves share about their own achievements. The development of problem-solving skills, critical and analytical thinking, and interdisciplinary and intersectional thinking skills are among the leading skills to which STEAM educational practices can make a major contribution.

Historical facts suitable for inclusion in the activities

Here are some historical facts from European history that can be incorporated into math lessons for primary classes:

The construction of ancient Roman buildings: Students can explore the mathematics behind the architecture of ancient Roman buildings, such as the use of symmetry and geometric shapes. They can measure and draw different shapes found in Roman architecture, such as circles, rectangles, and triangles, and calculate their areas and perimeters.

The development of the metric system: Students can learn about the history and adoption of the metric system, which originated in France. They can engage in measurement activities, converting between different metric units (e.g., centimetres to meters, grams to kilograms) and exploring the practical applications of the metric system in everyday life.

The exploration voyages of European explorers: Students can study the voyages of explorers like Christopher Columbus and Ferdinand Magellan. They can analyse and interpret maps, identify and calculate distances between various destinations, and practice map scale conversions to understand the challenges and distances faced by these explorers.

The development of clocks and timekeeping: Students can learn about the evolution of timekeeping devices, from ancient sundials to mechanical clocks. They can practice telling time using analogue clocks, explore the concept of 24-hour time, and engage in activities involving time calculations and problem-solving.

The use of Roman numerals: Students can explore Roman numerals, which were used in ancient Rome and are still encountered in various contexts today. They can learn the basic Roman numeral symbols and practice converting between Roman numerals and Arabic numerals. They can also engage in activities such as solving Roman numeral puzzles or creating their own Roman numeral charts.

The history of currencies: Students can learn about the development of different currencies in Europe over time. They can explore the value of historical coins and banknotes, practice currency conversions, and engage in activities that involve adding, subtracting, or calculating with different denominations.

The Renaissance and artistic symmetry: Students can study the concept of symmetry in art and architecture during the Renaissance period. They can learn about mirror symmetry, rotational symmetry, and bilateral symmetry in famous artworks, and engage in symmetry-related activities like drawing and identifying symmetrical shapes.

The study of population growth and change: Students can examine population data and explore the concepts of population growth and change in different European countries over time. They can analyse graphs and charts, calculate growth rates, and engage in activities related to interpreting and representing population data.

The history of European rulers and their reigns: Students can explore timelines of European rulers and their reigns, studying the duration of different dynasties or periods. They can practice working with dates, calculating the lengths of reigns, and engaging in activities that involve sequencing and ordering historical events.

The history of famous European mathematicians: Students can learn about influential European mathematicians throughout history, such as Pythagoras, Euclid, or Isaac Newton. They can explore the mathematicians' contributions and engage in activities that highlight the principles and concepts developed by these mathematicians, such as the Pythagorean theorem or geometric proofs.

Here are 12 specific ideas for applying manipulations during math lessons

1.Medieval fortress Baba Vida, Bulgaria:

- History and facts

- Making a model

- Mathematical concepts - Geometric figures

Figure 3 Source: 27.05.2023, https://pixabay.com/da/photos/vidin-bulgarien-f%C3%A6stning-baba-vida-2710276/

2. Magura Cave, Bulgaria:

- History and facts

- Cave paintings

- Mathematical Encyclopedia - Caves in Bulgaria

- Mathematical concepts - units of time, application of algorithms for addition and subtraction of numbers up to 1000, geometric figures

Figure 4 Source: 17.05.2023, Magura cave 019 - Магура – Уикипедия (wikipedia.org) https://bg.wikipedia.org/wiki/%D0%9C%D0%B0%D0%B3%D1%83%D1%80%D0%B0#/media/%D0%A4%D0%B0%D0%B9%D0%BB:Magura_cave_019.jpg

3. The Pirate Town of Omis, Croatia

- History and facts

- Pirate ship - project

- Mathematical concepts - units of measurement - length measurement, plane geometric figures, mass,

Figure 5 Source: 27.05.2023, https://pixabay.com/da/photos/mimice-omi%C5%A1-dalmatien-kroatien-hav-5002058/

4.The old town of Dubrovnik, Croatia

- History and facts

- Modelling with plasticine - model (DIY)

- Mathematical concepts - units of measurement, comparing numbers, geometric shapes

Figure 6 Source: 27.05.2023, https://pixabay.com/da/photos/kroatien-dubrovnik-503170/

5. European Parliament, Belgium

- History and facts - European Union

- European Parliament

- "The Great Square" - carpet weaving

- Heart of Europe sculpture - national flags

- Maths concepts - geometric shapes, word problems, multiplication property

Figure 7 Source: 27.05.2023, https://pixabay.com/da/photos/parlament-europ%C3%A6isk-strasbourg-1564430/

6. Elizabeth (Big Ben) Clock Tower, UK

- History and facts

- Clock Tower - Bell Tower Project

- Maths concepts - geometric shapes, word problems, units of time

Figure 8 Source: 27.05.2023, https://unsplash.com/photos/MdJq0zFUwrw

7. Treasures of Bulgaria

- History and facts

- Varna Treasure, Thracian Treasure, Panagyurishte Treasure, The Rogozen Treasure, Valchitrun Treasure

- Mathematical concepts - addition and subtraction of numbers up to 1000, geometric figures, word problems, modelling with numerical expressions

Figure 9 https://www.europeana.eu/bg/item/2021502/jspui_handle_pub_70

8. Ancient Rome, Italy

- History and facts

- Aqueducts, Sanitation and Water Supply in Ancient Rome

- The movement of water - working with waste materials, design

- Mathematical concepts - units of measurement, word problems, geometric figures,

Figure 10 Source: 27.05.2023, https://upload.wikimedia.org/wikipedia/commons/9/90/Roma-parco_degli_acquedotti03.jpg

9. Louvre, France

- History and facts

- Pyramid model - Pascal's triangles

- Art through the eyes of the artist - painting, sculpture, printmaking, etc.

- Mathematical concepts - Addition and subtraction of numbers up to 1000, Multiplication of numbers up to 1000, Geometric figures

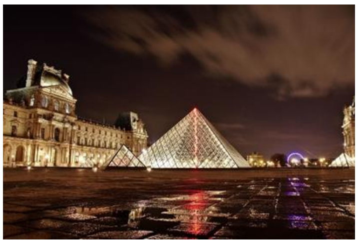


Figure 11 Source: 27.05.2023, https://pixabay.com/da/photos/lameller-paris-frankrig-arkitektur-1868203/

10. Tower of Pisa, Italy

- History and facts

- Model of a leaning tower - cardboard rolls and plaster mix

- Mathematical concepts - units of length and mass, types of angles, angle measurement, addition and subtraction without passing

Figure 12 Source: 27.05.2023, https://unsplash.com/photos/QL59SX34Mb8

11. Vitruvian man, Italy

- History and facts

- Mathematical concepts - halves, thirds, fourths, tenths, addition and subtraction to 1000, multiplication and division of numbers to 1000 by a single digit number, comparing numbers, extracting information from different sources and plotting in a table.

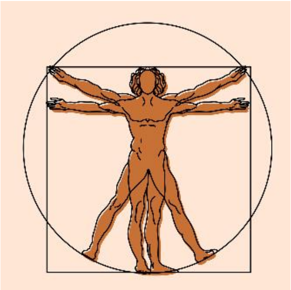


Figure 13 Source: 27.05.2023, https://pixabay.com/da/vectors/vitruvianske-mand-human-da-vinci-7212326/

12. Ancient Greece

- History and facts

- Timeline

- Acropolis - Ancient Greek columns and vases

- Mathematical concepts - drawing triangles on given vertices, units of measurement - century.

Figure 14 Source: 27.05.2023, https://pixabay.com/da/photos/akropolis-athen-gr%C3%A6kenland-gammel-2725910/

Which mathematical concepts can be represented by facts from European Historical Heritage?

Our goal is to use math in STEAM projects to achieve a more complete understanding of standard curriculum material. Through a variety of technical challenges, we strive to achieve a broader and deeper understanding of mathematical knowledge to describe situations in the surrounding world with a mathematical model, and extract information from a variety of sources.

In the early grades of mathematics education, some of the following mathematical concepts, which are related to European historical heritage, can be represented:

Number systems: In the early grades, the decimal number system is studied, which has its roots in Indo-Arabic culture and was introduced to Europe through Arab scholars in the Middle Ages.

Roman numerals: The Roman numeral system used by Ancient Rome is part of European historical heritage. Learning about Roman numerals can be useful for developing skills in numerical manipulation and understanding the structure of numbers.

Geometry: The basic principles of geometry, including shapes, lines, angles, and symmetry, can be represented with corresponding examples from European art and architecture history. For example, students can study different shapes found in historical buildings, such as rhombus, rectangle, and triangle, from various historical periods.

Measurement and units: The various units of measurement used in everyday life and trade are connected to European history. For instance, the meter and kilogram are metric units that were standardized in France in the 18th century.

Time and calendar: The concepts of time and calendar that we study have roots in European history. For example, the Gregorian calendar, widely used around the world today, was introduced by Pope Gregory XIII in 1582 and is part of European historical heritage.

These are just some of the mathematical concepts that can be associated with European historical heritage and presented in the early grades of mathematics education.

1. How to approach mathematics and cultural heritage?

Mathematics and history are integrated rather than taught as separate subjects. They connect and apply scientific principles and knowledge with technology, engineering, the arts and mathematics to address the challenges we face and solve real-world problems through the creation of products, prototypes and models.

Mathematical skills are used to analyse, reason, argue, prove and interpret solutions to problems in different scenarios from Europe's historical cultural heritage. An environment is created in which the student is their own teacher, explorer, traveller and a love of learning is fostered.

The traditional approach of teaching individual subjects in isolation no longer meets the challenges of today's world, society and work environment, and teaching students how to pass exams in different areas is not enough today.

STEAM education offers an interdisciplinary, holistic approach to education that prepares students for further study as well as for their future careers. The STEAM framework not only teaches students how to think critically, solve problems, and use creativity, it prepares students to work in fields that are poised for growth.

In an ever-changing, increasingly complex world, it is more important than ever that young people develop the knowledge and problem-solving skills to make sense of information and know how to gather and evaluate evidence to make decisions.

The main benefits of quality STEAM learning in mathematics classes through the discovery of European cultural heritage are:

- STEAM learning is inspired by the real world;

- The knowledge children learn is taught in a way that is applicable to the world we live in;

- Every STEAM activity is adaptable and creative;

- Children with different interests, when placed in the same group, can work as a team and partner with each other in a meaningful way. Good communication and teamwork are among the main things that the innovative learning model leads to;

- All students have the freedom to think creatively and innovatively;

- Exploration is part of the teaching methods;

- Although students make quite a few mistakes, they are not punished. This is one of the positive things because in this way adolescent children are not demotivated but learn to accept failure;

- STEAM lessons also help to reduce stress and tension in the classroom;

- Awareness of the links between science and the humanities.

Figure 15 https://pixabay.com/da/photos/at-l%C3%A6re-skole-f%C3%B8rskole-b%C3%B8rnehave-3701963/

Modern mathematics has become overly abstract and fiendishly complex, with vast swathes of it unlikely ever to become accessible to all. The truth is, however, that every person in their everyday life, whether they realize it or not, is more or less a mathematician. Every time children judge which doughnut is bigger, when they divide a pizza into equal pieces, when they count their change or even look at the clock, they are entering into their function as an 'everyday mathematician'.

Mathematics is a science studying primarily, but certainly not exclusively, numbers, shapes, and existing patterns and regularities. It is in its nature a kind of study of every single aspect of life and our surroundings, finds application in our every intention and action, and provides a sure bridge by which we can reach every other doctrine.

Everything without exception in our actual physical reality can be expressed through mathematics - the mosaic in ancient Greece, the sketch of Vitruvian Man, the Thracian treasures of Bulgaria, and more.

Mathematics and history are very closely related, and deeply so. The fact that historical processes take place over time conditions the possibility not just of an external application of mathematics, of modelling particular aspects and phenomena, but of understanding historical existence mathematically.

One reason for the strained relationship with this science is rooted in the simple fact that its mastery is a palpably difficult and protracted process. This is precisely the reason why metamathematics is represented through historical cultural heritage. In this way, students will enrich their general understanding of historical objects and unconsciously grasp mathematical concepts.

Children have an innate curiosity about the world around them. It is this quality that is the best strategy we can use to help them develop a positive attitude towards mathematics and to communicate 'you' with it.

One of the most important reasons to study history is that the same mistakes of the past can be repeated many times. The role of history is to expose these mistakes so that they are not repeated again and again. It helps us develop the existential skills we need to be good and conscientious citizens of our country and the world.

The link between history and mathematics will introduce students to cultural customs and traditions, so that they will adequately appreciate intercultural differences and strive for the equitable development of the society in which they live.

The study of history provides clarity about where we have come from, how the past has shaped us as individuals, and how it can help shape our present and future. It develops in pupils' high moral and ethical qualities. This science helps them to have a balanced understanding of the past and present, and to have a clear and objective idea of when our ancestors did right and wrong.

Children are much more enthusiastic about learning new things when they are connected to the world they know - their daily activities, surroundings and interests. This applies to all subjects, including mathematics.

Stories can be a useful tool for teaching mathematics in primary school, helping students to become familiar with mathematical concepts and to relate them to real examples and interesting stories. Here are some ways in which story can be used:

Historical Context of Numbers: Tell students about the historical development of numbers and counting. For example, you could share interesting facts about ancient civilizations, such as Ancient Egypt or the Maya, and the ways they counted and used numbers. This will help students make sense of numbers as something important and widely used in history.

Geometry and Architecture: Show students how geometric principles were used in historical architectural structures such as the pyramids, the gardens of Versailles, or Chinese pagodas. Examine pictures or models and discuss the shapes, symmetries, and proportions used. This will help students relate geometric concepts to real objects and better understand them.

Historical characters and mathematics: Tell students about famous mathematicians or scientists who have left their mark on history. For example, you could talk about Pythagoras and his theorem, Archimedes and his mathematical discoveries, or Euclid and his Elements. Tell them about important contributions to mathematics and how they have been used in a historical context.

Problems and Problems in Historical Scenarios: Create problems and problems that are based on historical scenarios and require mathematical thinking. For example, you might pose a problem related to counting objects in the Egyptian pyramid complex or calculating proportions in the construction of ancient architectural structures. This will help students apply mathematical concepts to real situations and understand their practical use.

Visual materials and games: use visual materials such as illustrations, pictures or animations that reflect historical aspects of mathematics. Make games and fun activities that are inspired by historical periods and require math skills. This will engage students and motivate them to have fun while learning mathematics.

- By depicting, modelling or making a model of a Medieval fortress, they will practise and apply their knowledge of geometric shapes and finding a circumference.

- By measuring quantities of materials and mixing them proportionally they will get to the secret of the Leaning Tower where they will apply their knowledge of angle measurement.

- They will discover the circumference and correct proportions in Leonardo Da Vinci's Man sketch.

- The fabulous stories about the creation of historical landmarks will take students through math problems with ease.

By using history in elementary school math instruction, you can introduce variety and interest to the subject and help students connect math concepts to historical context and real-world applications. They will apply the mathematical knowledge they have learned in practical activities. In this way, students will learn how someone was able to conquer the entire world and practice their mathematical knowledge by mathematizing history.

Younger children are used to shying away from developing new ideas as they are tied to taking risks. Adolescent boys and girls need to know that every new venture involves risks, but this should not frighten them but motivate them.

If they happen to fail at a STEAM project, they should try again. It is important to comment on all situations to avoid repeating mistakes. Students benefit from everything experienced.

2. European cultural heritage through STEAM in primary school

Why it is important to do this in primary schools as they are our target group?

A major challenge for modern education is to create a desire to learn. The ever-increasing amount of information that children have to absorb puts the education system and children under pressure with an unbearable amount of work that demotivates them for future learning. Yet this move fails to produce the necessary results in terms of knowledge and skills, mainly because information is not knowledge.

Children need to learn independently in and out of school, in and out of class. Combining different topics into one to solve a problem is both economical and highly motivating. Developing creativity creates lasting motivation for lifelong learning.

Creativity in STEAM allows the unity of knowledge to be realized and an interdisciplinary approach to be applied. This allows not only to save time and effort in learning, but also to create better, deeper knowledge as students create a unified model of the knowledge they acquire (through cross-curricular connections).

Children learn while they play from a very young age. Games develop many competences in children, for example teamwork, civic responsibility, emotional competence, creative and strategic thinking.

The nature and type of key competences activities in the curriculum point to the idea of their implementation through the application of the STEAM approach in primary education. Working in a unified system of the studied subjects and technologies leads to the creation of hybrid habits, among which the most rely on cooperation, creativity, communication and criticality.

Students apply and make sense of what they learn in a practical-applied way. Creating a common view of the content studied in the individual disciplines points towards integrating them into a global goal that implies the formation of key competencies in a real practical environment.

The strong interdisciplinarity is the simultaneous study of mathematics and natural, social and human sciences with all their interrelations and practical applicability. It contributes to multi-layered thinking and personality formation, as well as the integration of knowledge in all sciences - economic, social, natural, cultural, engineering, etc.

The creation of project-oriented products, models of historical landmarks, buildings, ecological systems, etc. will be linked to the acquisition and application by students of competencies in the various subjects studied, united in integrative learning.

Mathematical thinking can also be developed through play. In this way, children have the opportunity to explore mathematical principles in a fun and meaningful way, and the acquisition of new knowledge and skills is imperceptible.

Changing attitudes to mathematics require that we develop children's creativity, engage their attention, make learning fun and relate learning material to real historical objects. And the result is more than encouraging - inspired children with a great curiosity and desire for knowledge.

Building on pupils' strengths and developing their weaknesses helps them realise where they are and what they need to progress.

In mathematics, mistakes are perceived by students as the equivalent of a poor grade, failure in exams, or impending punishment. In real life, however, it is mistakes that are key to gaining life experience. Making mistakes and making sense of them is important for a deeper understanding of processes. Therefore, you can model the students' failures into opportunities to show them the value of them by having a constructive discussion through which they can reach the conclusions you want them to reach. To deal with students' fear of math tests, it is important to explain to them that it is normal to lose the thread, get it wrong, try again, get it wrong again, and finally succeed.

Students need to be encouraged to face challenges - difficult problems, projects or patterns that require a lot of knowledge, time, effort or creativity.

Thanks to this innovative learning method, young and old learn to accept failure, to think logically and creatively, and above all, not to give up. They work with passion and desire - the right formula leading to the goal and to success.

Why is STEM important for young children?

STEM is important in 21st century careers. In North America, for example, STEM careers are growing faster than any other profession. These careers typically come with higher-than-average incomes and play an important role in sustaining the economy. Science and technology have become very common in many occupations and in many new careers. The most important skills are the ability to acquire new knowledge quickly and to innovate.

STEM is fun. Instead of memorizing facts or having an instructed class, kids can play while they learn. Many educators believe that play equals learning, and STEM skills can be easily developed through fun, engaging activities.

Early use of STEM increases the passion for learning. When we expose young children to STEM ideas, we create a foundation for future passion and interest in these concepts, which can lead to continued learning and even successful careers in related fields.

Chapter 4 : Applying STEAM learning through pedagogical manipulation

After reviewing the state of mathematics education in Belgium, Croatia and Bulgaria, discussing the challenge of STEAM subjects and explaining how to integrate European history and cultural heritage, the next chapter will discuss using manipulatives in STEAM courses.

Using manipulatives is often recommended as an effective way of teaching mathematics (Carbonneau et al., 2016), but what exactly do we mean by "manipulative"? What are the benefits, how should they be integrated into our STEAM lessons, and are manipulatives still relevant for learning? There are many legitimate questions that this chapter will try to answer.

1. Manipulation - definition and brief history

Manipulatives can take many forms and are defined in many ways (Boggan, Harper & Whitmire, 2010). However, the common basis of the definition is: "physical objects used as pedagogical tools to engage students in hands-on learning of mathematics" (Using manipulatives cited in Boggan, Harper & Whitmire, 2010).

The different definitions are then enriched by the various benefits that research suggests it has. For example, Belenky et al. (2009, p2) add that "Manipulatives are intended to help the student concretise knowledge by expressing concepts and carrying out problem-solving steps using the object". Likewise, Lewis (2012, p. 1 cited in Cockett & Kilgour, 2015) states that "Manipulatives provide visible models that help students solve problems and develop concepts" (Lewis, 2012, p. 1 cited in Cockett & Kilgour, 2015)."

Since the beginning of civilisation, many countries and cultures have used objects to solve every day mathematical problems. Initially, the materials were simple and consisted of clay tablets and sand that they wrote in with their fingers (Boggan, Harper & Whitmire, 2010).

As different peoples' needs evolved, so did the materials they used. The Inca Quipu and the abacus have since appeared (Boggan, Harper & Whitmire, 2010).

It was not until the late 1800s that the first manipulatives were invented. They showed promising results for all learning styles and covered various mathematical concepts. Two major educators who worked with these tools were Maria Montessori and Friedrich Froebel (Boggan, Harper & Whitmire, 2010).

Manipulatives are still widely used today, but they have evolved to be more complex and comprehensive and can also be virtual. There is considerable research into their benefits and limitations (D'Angelo & Iliev, 2012).

2. Advantages

By its very essence, mathematics is an abstract concept. So, what are the advantages of including it in the experimental sciences (GRACOM, 2021)?

The numerous benefits of STEAM manipulations from cognitivist theories and our modern knowledge of human development (McNeil & Jarvin cited in Carbonneau et al., 2016).

Manipulations that have their roots in developmental theories and constructivism:

Some prominent educationalists specialising in developmental theories, such as Montessori (1964), Bruner (1964), and again Piaget (1962), agree that the use of manipulatives supports the development of abstract reasoning (Carbonneau et al., 2016). Therefore, much research has shown that children who have not yet reached the stage of abstract representation will benefit most from using manipulatives (Fennema, Resnick & Omanson, cited in Carbonneau et al., 2016). This result can be explained by the fact that young children need more physical interaction to construct meaning (Bruner, Piaget & Coltman, cited in Carbonneau et al., 2016). Shaw (cited in Cockett & Kilgour, 2015) also states that when students physically move the manipulatives to show the different relationships, their sense of touch is actively engaged, facilitating their understanding.

Piaget identified four primary stages of development: sensorimotor, pre-operational, concrete operational and formal operational (Ojose, 2008). In the pre-operational and concrete operational stages, manipulations will be most relevant for learning. Indeed, the pre-operational stage is characterised by increased linguistic abilities (with over-generalisations), symbolic thinking, an egocentric perspective and limited logic. Therefore, concrete manipulations and verbalisation can help teachers understand students' thought processes. According to Burns & Silbey (cited in Ojose, 2008), concrete manipulations are a perfect way to foster the development of the concrete operational stage. These manipulations make abstract concepts more concrete and thus enable problem-solving. The teacher's challenge is creating links between the manipulation and the idea (Burns & Silbey cited in Ojose, 2008).

Teachers who integrate manipulatives into their lessons create classrooms based on constructivism (D'Angelo & Iliev, 2012). This theory places students at the centre of learning, which they construct through their interactions with the environment (material and social). In constructivist mathematics classrooms, students are expected to be active in their learning. In this pedagogy, the teacher must carefully organise the teaching and materials to reach the students' zone of proximal development. Vygotsky defined this zone of proximal development as "those functions that have not yet reached maturity but are in the process of maturing" (cited by D'Angelo & Iliev, 2012, p. 4). This zone can be reached through the presentation of manipulatives, as learners will have to make the connection between their previous and new experiences.

Manipulations to give meaning to learning:

Another benefit of manipulatives is that they facilitate connections between abstract mathematical concepts and everyday life (Brown et al. Cited in

Carbonneau et al., 2016; Brown, Collins, & Duguid in Belenky et al. l., 2009; Hawkins, Boggan, Harper and Whitmire cited in Cockett, & Kilgour, 2015). Other research supports this finding as it has been shown that when a student has little or no prior knowledge of a topic, presenting a known concrete object can help the student create relevant thought patterns (Tindall-Ford & Sweller, cited in Carbonneau et al., 2016). Stein and Bavolino's research (cited in Cockett & Kilgour, 2015) came to the same conclusion: teachers create a more meaningful experience for students by providing manipulatives. Indeed, by providing them with a concrete form, they can perceive the usefulness of the mathematical concept in everyday life. Perceiving the meaning of the activity also increases learners' motivation (Cordova & Lepper, Schraw, Flowerday, & Lehman, cited in Belenky et al. l., 2009).

Manipulations to engage and motivate learners:

Another benefit of manipulations that follows from the previous one is that they allow for greater engagement on the part of the learners and enable them to be more focused on the task (Florence, 2012, cited in Cockett & Kilgour, 2015). Indeed, presenting learners with a task that they understand the purpose of and in which they are active will enable them to be more engaged, motivated and focused (Swirling, Moyer, cited in Cockett & Kilgour, 2015). Xie, Antle and Motamedi (cited in Cockett & Kilgour, 2015) go further in their research between manipulative use and the learning process as they linked enjoyment to engagement. Shaw (cited in Cockett & Kilgour, 2015) also suggests that using manipulatives can help students become more confident as they allow them to solve complex mathematical tasks.

Manipulations to memorise concepts:

Manipulations also make it possible to present several means of representation and consequently allow for better retrieval of stored information (Sweller, Merrienboer, & Paas cited in Belenky et al. l., 2009) since encoding has been able to take place through two channels; the verbal channel and the non-verbal channel (the motor channel) (Clark & Paivio cited in Carbonneau et al., 2016). Therefore, a student who has learned a concept through manipulations when retrieving the information from memory will have access to two types of information. Successful retrieval of one kind of information (verbal) will allow recovery of the other type of information (non-verbal) and thus result in better performance in terms of learning outcomes (Marley & Levin, cited in Carbonneau et al., 2016).

Manipulations to meet the needs of all learners:

Manipulations are helpful ways of meeting the different learning styles of learners. Indeed, they are appropriate for students with primarily a visual and/or kinaesthetic learning style (Sundstorm, cited in Cockett & Kilgour, 2015). However, manipulatives are not only useful for these students. Indeed, as explained in the previous paragraph, the presentation of different modes of mathematical representation benefits recall for all learners.

Research has also shown that manipulatives are recommended for low-achieving students or those with learning difficulties (Boggan, Harper & Whitmire cited in Cockett & Kilgour, 2015).

Another advantage of manipulatives that has not yet been mentioned in this chapter is that of interaction with peers. Indeed, immersion in manipulation will allow interaction between students; they will exchange their points of view and listen to different perceptions, enabling them to build their learning (D'Angelo & Iliev, 2012). According to research conducted by Moyer (cited in D'Angelo & Iliev, 2012), the discussion is part of the use of the manipulative and is considered an essential element in the effectiveness of the manipulative.

3. Conditions for implementation

Although manipulatives are often recommended as effective teaching strategies, the literature is not unanimous about their positive effects on learning. Therefore, other factors influence the effectiveness of manipulatives, such as the level of guidance provided by the teacher, the age of the learners, the type of manipulation or the learning environment itself (Carbonneau et al., 2016).

Smith (cited in Cockett & Kilgour, 2015, p. 4) states: "There are probably as many bad ways to teach with manipulatives as there are without".

Therefore, addressing the conditions for applying manipulatives seems essential to make them as relevant and effective as possible.

Much research has been done into the level of support a teacher should provide students during manipulative activities. The term "support" can refer to several types of teaching formats. It can refer to the interactions between the student and the teacher during the learning process (Terwel et al., 2009, Mayer cited in Horran & Carr, 2019); it can also refer to the scaffolding provided by the teacher by giving concrete examples or by appropriately ordering the challenges encountered (Baroody et al. 2015, Chen, Kalyuga, & Sweller cited in Horran & Carr, 2019). Finally, coaching here refers to student-teacher interactions (answering student questions, providing feedback, and guiding questions).

A recent literature synthesis on the subject suggests that a medium level of support leads to better results on information retention than unaccompanied discovery (Alfieri, Brooks, Aldrich, & Tenenbaum, cited in Carbonneau et al., 2016). Indeed, it is not enough to give students the material to be active. Being active in learning has several advantages. It facilitates learning by doing (Anzai & Simon cited in Belenky et al. l., 2009) and increases attention and engagement (Chi cited in Belenky et al. l., 2009). For students to be genuinely active and engage in deeper learning processes, the teacher can provide support by asking metacognitive questions (Chi, Graesser & Black cited in Belenky et al. l., 2009). These metacognitive questions will help students to reflect on different aspects of the material and to find solutions to the problem at hand (Schoenfeld, cited in Belenky et al. l., 2009).

However, other research complements this conclusion by adding that too much guidance should not be provided at the risk of creating learning that is not transferable to different contexts (Martin, cited in Carbonneau et al., 2016).

However, Horran & Carr's 2019 study concludes that the level of support provided will depend on the learning objective and the audience. Indeed, a high level of support would allow for better retention of information, while a low level of support would allow for a more effective transfer of learning (Carbonneau, Marley, and Selig, cited in Horran & Carr, 2019).

The teacher's support is not the only factor influencing the effectiveness of manipulatives on learning; the material used also plays a significant role (Belenky et al., 2009).

While the authenticity of the material can be an advantage (it allows students to contextualise learning), it can also be a disadvantage. Indeed, material and situations that are too realistic can make the transfer of learning more complicated (Goldstone & Sakamoto, Son & Goldstone, cited in Belenky et al. l., 2009). In addition, material that is too detailed or has too many superfluous elements can distract students from the essential features (Harp & Mayer, Son & Goldstone, cited in Belenky et al. l., 2009).

Thierry Dias in his book "Manipuler et expérimenter en mathématiques" (cited in Gracom, 2021), details 4 phases to set up manipulations in a relevant and efficient way for learning.

Phase 1

The first phase consists of allowing time for the pupils to apprehend the material freely. This phase consists of observation, manipulation and experimentation.

Phase 2

The second stage is a debriefing phase where pupils explain what they have done and observed in the previous step. At this stage, all reflections are accepted, and the teacher provides a supportive framework and can stimulate the discussion with questions if necessary.

Phase 3

This phase involves sorting out the approaches or explanations made earlier. This phase will lead to debate, argumentation and validation. The teacher's role is to lead the debate and refocus the discourse if necessary.

Phase 4

The last stage will allow the institutionalisation of the concepts discovered. The aim is to provide an official record of the learning found to all pupils.

Dias concludes by making three recommendations so that the carrying out of manipulations enables pupils to achieve abstraction.

Manipulation should:

- Bring out a questioning process;
- Create a restriction = the manipulation becomes the only way to solve the problem;
- Be considered in a complete sequence that ends with a verbalisation of the mathematical notions learned.

The next point addresses examples of good practice. This point is useful for providing concrete examples of how STEAM has been approached throughout history.

4. Good practices

Examples of manipulations in STEAM classes can be found in other Erasmus + projects, such as the STEAM builders project, but also in the museum's school service. Some museums provide manipulation to understand their content better, but there is also mathematics museum who become more and more popular. Some offer simple manipulation for the primary level, and some even go to high education level by providing manipulation to understand statistics.

The STEAM Builders project:

The STEAM Builders project is an Erasmus+ project that promotes a hands-on non-formal approach to STEAM through the recreation of historical manipulations to boost STEAM interest for 10 – 15-year-olds.

As planned in our STEAM in Times project, plans for manipulations have been created, as well as teaching materials to exploit these manipulations.

Case 1: Creating a Sextant

During the 2020-2021 school year, 42 pupils from a secondary school in Greece tested the manipulator on the creation of a Sextant.

The teaching sequence lasted a total of 3 hours.

The plan involved creating the sextant, and the teaching sequence included using the sextant to measure the height of their school.

Using a learning-by-doing approach, the students

- Made the sextant using the plan;
- With the teacher's help, learned about Thales and his theorem on similar triangles;
- studied Xenagoras (2nd century BC), who based his research on Thales' theorems, calculated the height of the summit of the Greek mountain of Western Olympus, called Flambouros, which is the highest peak in the world.
- and finally, they measured the height of their school as a group by doing the the necessary calculations.

After the activity, the students were asked to answer a questionnaire to find out what they thought of the sequence they had just completed. The questionnaire showed that they had all really enjoyed the learning experience thanks to the manipulations.

This first example is a demonstration of the successful integration of STEAM subjects through manipulations in secondary school.

A second example from the STEAM builders project is the creation of glass windows.

Case 2: Designing a stained glass rose window

One of the project partners organised workshops across the city to show that using local heritage to learn STEAM is possible.

One of the workshops involved walking through the town streets to admire the 14th-century Gothic church of Notre Dame de l'Assomption in Beaumont de Lomagne, France. The mediators draw the children's attention to one of the stained glass windows on the outside façade. They then enter the building to take a closer look.

The mediator then explains to the children how a stained-glass window is made: the choice of model and the making of the model are the first stages. Next, the master glassmaker represents the design, the colours, the lead links and the metal frame, all in life-size. The glass is diamond-cut, and each piece is painted with glass paint and fired. All the pieces are assembled and joined with lead. The resulting glass roof is then secured with a metal frame.

The tour ends, and the group returns to the centre to create their own rosette. Using the various tools at their disposal - compasses, rulers, protractors -, the children are invited to become creators in their own right. Combining symmetry and geometry, they reproduce the model suggested by the guide: the church and its stained-glass window come to life. All that remains is to transfer it onto transparent paper and illuminate it in the style of a master glassmaker.

Thanks to this workshop, children have the chance to learn about a heritage, an ancestral technique and an art form, and to play at being an artist in their own right - to become a master glassmaker!

Museums of mathematics:

Museums in general bring many benefits to be integrated into a learning sequence. Museums of mathematics are particularly interesting for our project, as they very often offer hands-on activities to help us understand subjects related to mathematics and STEAM.

Case 1: Kaleidi - Belgium

Kaleidi is a not-for-profit organisation that aims to increase knowledge of mathematics and numeracy by developing a passion and enthusiasm for mathematics and numeracy.

The association offers mathematical exhibitions on various subjects, visits schools and provides training for teachers.

Kaleidi's Math exhibition introduces visitors to the concept of tessellation and transformation of the plane using a puzzle made from kangaroos, to the concept of probability using dice rolls and to how Archimedes discovered the area of a disc.

Case 2: Mathematikum - Germany

The Mathematikum in Giessen is the world's leading mathematical science centre.

The museum offers over 170 exhibitions for visitors of all ages and levels of education. Through these exhibitions it is possible to become an old master builder and create an arch that is both delicate and extremely robust, to try to rebuild Leonardo da Vinci's bridge by creating a solid bridge without nails, glue or other tools.

In conclusion, this chapter has shown us the many advantages of manipulatives for learning, such as giving meaning to learning, improving recall, increasing motivation and commitment, and meeting all learning styles. Suggestions for the effective use of manipulatives were also put forward, ending with examples of good practice.

Chapter 5 : Project activities

1. Preliminary preparation, description and structure of drawings, materials, exercises, and 3D projects included in STEAM lessons

1. Medieval fortress Baba Vida, Bulgaria:

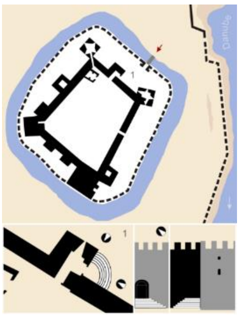


Figure 16 Source 23.05.2023, Baba Vida - Баба Вида – Уикипедия (wikipedia.org)

- Observation and analysis of the medieval fortress "Baba Vida".

- Discussion of the whole and parts of the whole.

- Drawing in a square grid, preparing the necessary materials and making a 3D project.

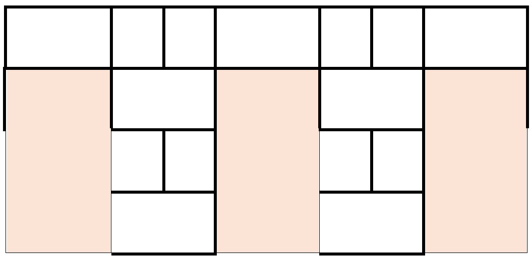


Figure 17 (The idea is adapted from page 49 of the presentation : Source 23.05.2023, https://shorturl.at/ayzM5)

-Making a model of a medieval dwelling

Example: [Video spell - workshop " Kak that you are i do model on prehistoric housing " . - YouTube]

2. The Pirate City of Omis , Croatia

- History of Pirate Town - introduction to the characteristic features

- Discussing options and generating ideas for creating a vessel

Figure 18 Source 23.05.2023, TO MAKE MYSELF SAL! • Moe mentally bebe (umnobebe.com) (Let's make a raft! My smart baby) https://umnobebe.com/2016/05/11/%D0%B4%D0%B0-%D1%81%D0%B8-%D0%BD%D0%B0%D0%BF%D1%80%D0%B0%D0%B2%D0%B8%D0%BC-%D1%81%D0%B0%D0%BB/

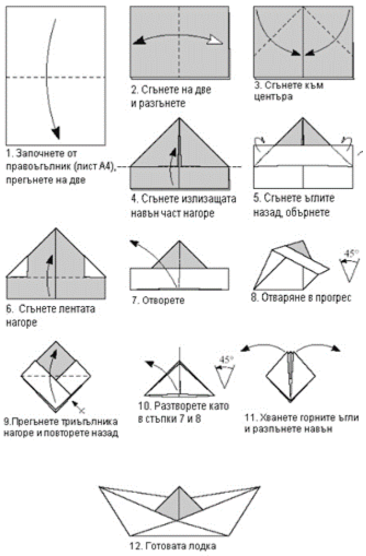


Figure 19 Source 23.05.2023, https://az-deteto.bg/kak-da-si-napravim-lodka/9635/view.html (How to make a boat)

- Establish and test the stability of the structure with objects of different weights and sizes

Here is a sample: http://krokotak.com/2012/06/lodka-ot-korkovi-tapi-ili-domashno-prigotvena-morska-bura/?fbclid=IwAR1AfeTfCXQCZauOtVN9s4UARZsO4-6_NgS672hlzpB8XchsMM2k2VoSHRc

Figure 20 Source: personal archive

3. The Tower of Pisa, Italy

- Introduction to the history of the creation of the Tower

- Making a model from cardboard rolls and measuring the angle of inclination

Figure 21 Source 05/23/2023, Pinterest https://www.pinterest.com/pin/326229566773972137/sent/?fsf=&invite_code=59e8305a45cf4c1c8195860a0100c45c&sender=307652355703232135

- Preparation and casting of gypsum mixture in the finished model

Figure 22 Source 23.05.2023, krokotak | TEBESHIRI FOR RIVANAN - make you are sam (krokotak.com) – (krokotak | Drawing tutorials - do it yourself) https://krokotak.com/2010/06/da-si-napravim-sami-tebeshiri-za-risuvane/?fbclid=IwAR2kwqR0j9xd50-UBG1dqScVYYpsyvTDoB6wZ1owU2oTYQtHnBfVx0PzGWk

- Create chalk and practice knowledge of angles and types of angles

Figure 23 https://pixabay.com/da/photos/gadekridt-street-art-kinderbild-625217/

4. Magura Cave, Bulgaria

- History and facts - The secret messages of antiquity in Magura Cave

Figure 24 https://en.wikibooks.org/wiki/The_cave_painting_in_Magura_Cave

- Comparing units of measure, extracting information from text and structuring in a table

- Cave drawings - charcoal drawing

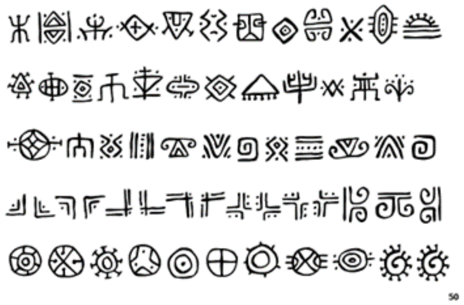


Figure 25 Source 23.05.2023, CavemanDingbatsTwo.gif (864×576) (identifont.com) http://www.identifont.com/samples2/corradine/CavemanDingbatsTwo.gif

Making medieval tools

[Prehistorians with ki blades - YouTube (Prehistoric tools - The video Shoo in Bulgarian , thigh contains mostly images and can be used in training in other languages).]

Figure 26 Source 23.05.2023, Big Stone Ax - Official Scum Wiki (fandom.com) https://scum.fandom.com/wiki/Big_Stone_Axe?file=Improvised_Large_Stone_Axe.png

5. Ancient Greece, Greece

- History and Facts - Architecture of the Acropolis

https://www.youtube.com/watch?v=ulAxMLJ7O7M (Virtual tour in ancient Athens (5th century BC) - 3D reconstruction)

- Time Line - Century

- Ancient Greek mosaic - Virtual they studied ŝ en cabinet per mathematics (cabinet.bg) (Virtual school cabinet for mathematics - in Bulgarian)

- Mirror image - symmetry - model of pavement with natural materials

Figure 27 Source 23.05.2023 https://www.pinterest.com/pin/687320185911/

- Making a math column with tasks

Figure 28 Source 23.05.2023 https://www.pinterest.com/pin/242069167429565891/

Method of manufacture: Cup Equations Math Activity for Kids - YouTube

6. Treasures of Bulgaria

- History and facts - Secrets at a on Bulgaria - YouTube (The treasure of Bulgaria - the video clip Shoo provided with English subtitles)

Varna Treasure, Thracian Treasure, Panagyur ishte Treasure, Rogozen Treasure, Valchitrun Treasure

- Making a figure from salt dough

Here is the recipe for salty dough:

1 c. cup flour, 1/2 cup salt, 1 tbsp. spoon olive oil, 1/2 c. a glass of water.

Bake/dry in a low oven or microwave. Color after they have cooled down. When making forms for coloring, make sure not to make them too thick, because moisture can remain in them and they become soft.

You can see the images here: crocotak | RE C EPTA for Salted testo (krokotak.com) (Krokotak – Recipe for salty dough) – geometric shapes, measurement units

Figure 29 Source: 23.05.2023 https://www.pinterest.com/pin/302592474133505/

7. Old Town of Dubrovnik, Croatia

- History and facts - Dubrovnik in 4K - YouTube

Figure 30 https://pixabay.com/da/photos/kroatien-dubrovnik-springvand-238006/

- Modeling with plasticine and waste materials - making a model of a fountain
- measurement units, comparing numbers, geometric figures

Sample ideas for making a fountain: How to Make a Model Human Heart - YouTube (103) Pinterest

8. European Parliament, Belgium

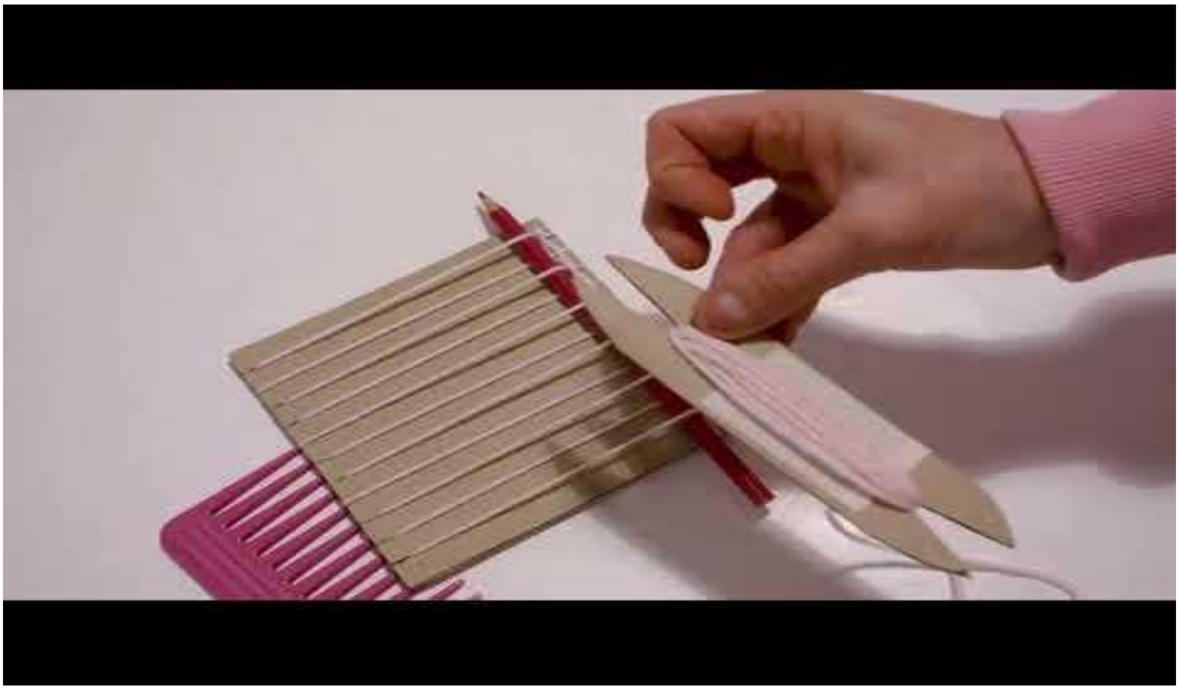
- History and facts - European Union creation

-European Parliament

- "The Big Square" - carpet weaving - text problems, multiplication property

Figure 31 Source: https://www.wallpaperflare.com/grand-place-brussels-belgium-travel-europe-landmark-architecture-wallpaper-akfie

[СТАН. Преплитане на нишки в ръчно изработен стан. Килими]



(Loom. Weaving threads in a handmade loom. Carpets)

Kilimce from scraps (ot-nishto-neshto.blogspot.com) (Rag carpet - in Bulgarian)

- Sculpture "Heart of Europe"

- national flags - geometric figures

Figure 32 https://freesvg.org/european-union-flags-ii

9. Elizabeth (Big Ben) Clock Tower, UK

- History and facts

- Clock Tower - Bell Tower Project

Source: personal archive

- Making a clock tower and a bell tower - geometric figures, word problems, time units

10. Louvre, France

- History and facts - [Louvre | Miracles n a Europe (wonders-of-europe.com)](#) [In Bulgarian](#)

Virtual Tour - [Online tours (louvre.fr)](#)

- Pyramid model - Pascal's triangles

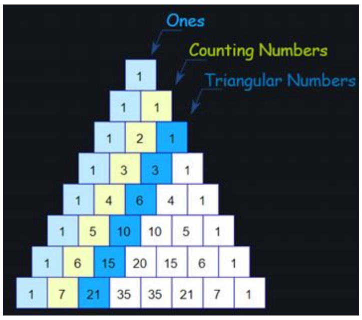


Figure 33 Source: 23.05.2023,: https://www.mathsisfun.com/pascals-triangle.html

- Art through the eyes of the artist - painting, sculpture, graphics, etc.

- Mathematical concepts - Addition and subtraction of numbers up to 1000, Multiplication of numbers up to 1000, Geometric figures

11. Ancient Rome, Italy

- History and facts - [Interesting ! Roman culture . History 5th class (ucha.se)]

- Aqueducts, sewers and water supply in Ancient Rome

Figure 34 Source: 23.05.2023, Rimskite aqueducts — miracles _ on engineering (jw.org) (Roman aqueducts - wonders of engineering)

- The movement of water - working with waste materials, project

Figure 35 Source: 23.05.2023, Educating Casia . The Aqueduct

- Mathematical concepts - measurement units, word problems, geometric figures, addition, and subtraction

12. Vitruvian Man

-History and facts Da Vinci's Vitruvian Man of math - James Earle - YouTube

A conceptual option for recreating a Vitruvian man How to draw Leonardo Da Vinci's Vitruvian Man Real Easy - Step by Step - YouTube

- Making a Robot showing parts of the whole

Figure 36 https://www.pinterest.com/pin/703756184849980/

Instructions for work: Fraction Robot || Fraction Activity for Grade 3, 4, 5, 6 - YouTube

Conclusion

The "Manipulating Math through History" guide is the first written result of the Erasmus + STEAM in Times project, a collaboration between Bulgaria, Belgium and Croatia.

The project was initiated in response to the low results achieved by European pupils in mathematics and science. Indeed, the PISA (2018) studies have shown that the EU education system is still lacking in the STEAM field. The results show that 22.4% of European pupils perform poorly in mathematics and 21.6% in science.

One in five young people in Europe lacks the skills essential for many valuable jobs in today's economy. What's more, current studies show that 65% of children growing up today will be working in a job that doesn't yet exist. It is, therefore, essential to give today's young people the skills they need to integrate and participate actively in tomorrow's world.

STEAM stands for Science, Technology, Engineering, Arts and Mathematics. The teaching method for these disciplines is interdisciplinary. The aim is to connect these different subjects to contextualise learning, give it meaning and enable pupils to decompartmentalise their education. Adding the letter 'A' to the term brings a creative dimension to learning, includes all learners and encourages problem-solving.

STEAM learning activities also lend themselves to the inclusion of European historical facts. Indeed, integrating a historical dimension into STEAM learning makes it possible to contextualise learning and thus give it meaning, increasing pupils' commitment and motivation to the task.

Our project looks at STEAM teaching through history using manipulatives. Educational manipulatives offer several advantages, such as more outstanding student commitment, better access to abstraction and better retention of information if they are appropriately integrated into the teaching. These manipulatives will also enable us to teach students differently by making them active participants in their own learning. With this method, we're not just teaching information but skills that are useful in everyday life. These skills will be particularly in demand in the professions of tomorrow. These experiments, in which the subject areas are decompartmentalised (STEAM and history), will enable the pupils to develop their curiosity, which will help them to develop learning methods and to discover their interests and passions.

The innovative and attractive material and the method used are intended to be as inclusive as possible so that all pupils, whether they have learning difficulties, a learning disability or are behind in learning STEAM subjects, can be considered.

The STEAM in Times project will offer 36 construction plans for teaching manipulatives, each accompanied by a teaching sequence to help teachers integrate them into their lessons.

We hope that this guide and the material that follows will be useful to you in setting up STEAM activities in your lessons. Although we don't know what tomorrow's jobs will be, we do know that they will involve STEAM-related skills, so we might as well get started as soon as possible!

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